Om tat sat

Does P=NP?

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***Abstract*-The P=NP? problem was emerged in 1971. This is one of the longstanding unresolved problems in computer science. Its solution is directly related to the determination of time complexity of NP-complete problems.**

The purpose of this work is to provide the solution of P=NP? problem.

KEY WORDS: algorithms and complexity, timetable scheduling, computer science.

I. INTRODUCTION

The Peaceful Coexistence Algorithm (PCA) is introduced in this work. PCA is a polynomial time algorithm that the General Timetable Problem (GTP) solves for any instances. The GTP is NP-complete. This means, that each problem in NP may be solved in polynomial time, including 3SAT problem.

II. FORMULATION OF GTP

As the matter of fact, the problem itself is very versatile and complex, that is why, for the sake of simplification a certain secondary school is observed as an object of implementation.

In reality, the generality is kept, because all ideas, denotations, considerations done for the school, in essence are the same for the general problem and may be generalized without any difficulty, if it is necessary.

Assumptions

In school, there are N_c classes, N_r classrooms, N_t teachers and N_s subjects are taught.

In school, there are ten different level classes {1,2,3,4,5,6,7,8,9,10} and a few number of classes of the same level are possible. The numbers from 1 to 10 and English letters are used in order to construct additional codes(AC) for identification of all classes (e.g. if there are 3 of the first level classes and 4 of the second level classes and etc., then their addidional codes are 1A, 1B, 1C; 2A, 2B, 2C, 2D; … etc.).

Each subject of N_s subjects in school is to be taught in a separate classroom.

Each teacher in school teaches only one subject.

Each subject is considered to be either Sharing or Unsharing.

Any Sharing subject is to be taught by two teachers in two subgroups of the same class in two separate classrooms.

Sharing teachers is a group of exactly two teachers in a class, who teach the same Sharing subject in the class.

Any Unsharing subject in a class is to be taught by one teacher only.

Any Sharing subject in a class is to be taught by a group of Sharing teachers only.

The start up time of all first lessons of a class is the same every day.

The duration of any lesson is considered to be one hour.

A. Input Data Variables

The following variables are used for the Input Data of GTP:

c,r,t,s,d,l - lower case indexes of entries of arrays, which are usually used as codes of a class, classroom, teacher, subject, day and lesson respectively.

N_c - the number of classes in school.

N_s - the number of subjects in school.

N_g=2- the number of subgroups into which a class is to be split if a Sharing subject is taught.

N_t - the number of teachers in school.

$S=\{S_s/s=1,\dots,N_s\}$ - the set of subjects taught in school, where S_s is the name of subject s and index s is the code of the subject.

$C=\{C_c\ /\ c=1,\dots,N_c\}$ - the set of classes in school, where C_c is AC of a class and index c is the code of the class.

$Q^+=\{q^+_c\ /\ c=1,\dots,N_c\}$ - q^+_c is the total quantity of lessons of class c to be timetabled for a week.

$D^o=\{d_c\ /\ c=1,\dots,N_c\}$ - d_c is the number of days, that can be timetabled for the lessons of class c for a week.

$B=\{b_c\ /\ c=1,\dots,N_c\}$ - b_c is the first hour, that must be timetabled for the lessons of class c every day for a week.

$E=\{e_c=b_c+ceil(q^+_c/d_c)-1\ /\ c=1,\dots,N_c\}$ - e_c is the last possible hour, that can be timetabled for the lessons of class c every day for a week (see Appendix).

$F=\{f_c=q^+_c - floor(q^+_c/d_c)*d_c\ /\ c=1,\dots,N_c\}$ - f_c is the number of last hours that can be timetabled for the lessons of class c for a week (see Appendix).

$N_d=\max\{d_1,d_2,\dots,d_{Nc}\}$ - the maximum number of days, that can be timetabled for the lessons of school for a week.

$N_l=\max\{e_1,e_2,\dots,e_{Nc}\}$ - the last hour, that can generally be timetabled for the lessons of school every day for a week

$T=\{T_t\ /\ t=1,\dots,N_t\}$ - the set of teachers in school, where T_t is the name of teacher t and index t is the code of the teacher.

$Q=\{q_{c,t,s}\ /\ c=1,\dots,N_c,\ t=1,\dots,N_t,\ s=1,\dots,N_s\}$ - $q_{c,t,s}$ is the quantity of lessons of subject s to be taught by teacher t in class c, such that the following equality holds (see Appendix):

$$q_c^+ = \sum_{t=1}^{N_t}\sum_{s=1}^{N_s} q_{c,t,s} - \left(\sum_{t=1}^{N_t}\sum_{s=1}^{N_s} (q_{c,t,s} * IsSh(c,s))\right)/N_g .$$

N_q - the total quantity of lessons of school:

$$N_q = \sum_{c=1}^{N_c} q_c^+ .$$

B. Output Data Variables

$Z=\{z_{c,l,d}$ / $c=1,\ldots,N_c$, $l=1,\ldots,N_l$, $d=1,\ldots,N_d\}$ - the timetable of the school for a week where $z_{c,l,d}$ is the entry of Z. At the beginning all entries of Z are 0. After constructing Z, if $z_{c,l,d}$ is not zero, then it is the code of a subject, that is to be taught by corresponding teachers in class c at hour l on day d.

$Z_c=\{z_{c,l,d}$ / $l=1,\ldots,N_l$, $d=1,\ldots,N_d\}$ - the timetable of class c for a week.

C. Decision problem of GTP

Form the school's timetable, by timetabling the entries of Z for the lessons of all subjects so that no overlapping exists: subject to(see Appendix):

No overlapping constraint for any fixed values of t,l,d:

$$\sum_{c=1}^{N_c}\{t\in T(c,l,d)\ ?\ 1\ :\ 0\}\le 1;$$

Lessons constraint:

$$\sum_{c=1}^{N_c}\sum_{d=1}^{d_c}\sum_{l=b_c}^{e_c}\{z_{c,l,d}\ne 0\,?\,1:0\}=N_q.$$

All the variables are integers.

III. SOLUTION OF GTP

The solution of GTP is the PCA. The PCA realizes in iterations. Some basic ideas and denotations are needed for PCA.

A. Definitions

Definition 1. An entry $z_{c,l,d}$ of Z, for any values of c, t and s, is called Accessible Entry (AE) for subject s, if $q_{c,t,s}\neq 0$, $z_{c,l,d}=0$ and

$$\sum_{c^o=1}^{N_c}\{ShT(c,t,s)\cap T(c^o,l,d)\ \neq\ \varnothing\,?\ 1:0\}=0,$$

that is, this entry can be timetabled for subject s(see Appendix).

Definition 2. A subject is called Subject Accessible to entry $z_{c,l,d}$ of Z, if this entry can be timetabled for that subject.

Definition 3. An entry $z_{c,l,d}$ of Z is called Accessible Entry (AE), if this entry can be timetabled for at least one subject.

Definition 4. The number $X_{c,l,d}$ of all subjects accessible to AE $z_{c,l,d}$ is called height of AE $z_{c,l,d}$.

Definition 5. $Z_{l,d}=\{z_{c,l,d}$ / $c=1,\ldots,N_c\}$is called Pivot Entry (PE) of Z, where (l,d) is the coordinate of PE $Z_{l,d}$ and (c,l,d) is the coordinate of entry $z_{c,l,d}$.

Definition 6. $Z_{l,d}$ is called Accessible PE (APE), if it has at least one accessible entry.

Definition 7. $h_{l,d}$ is called height of APE $Z_{l,d}$. Its value may be found by the following way:

$$h_{l,d}=\sum_{c=1}^{N_c}X_{c,l,d}.$$

Definition 8. A number of coordinates of APE of Z which are grouped by a common principle is called Accessible Pivot Domain(APD). Z must be split into APD, such that $D=\{D_{[m]}$ / $m=1,\ldots,N_D\}$, where N_D is the number of APD of Z and [m] is the coordinate of APD $D_{[m]}$.

Definition 9. $D_{[m]}=\{[m,j]=(l_{m,j},d_{m,j})$ / $j=1,\ldots,n_m\}$, where n_m is the number of APE in $D_{[m]}$ and [m,j] is the short cut of the coordinate of APE j in $D_{[m]}$.

Definition 10. $D_{[m]}=\{$ $D_{\lfloor c,m\rfloor}$ / $c=1,\ldots,N_c\}$, where $D_{\lfloor c,m\rfloor}$ is called class APD and contains all AE of $D_{[m]}$ in class c.

Definition 11. $D_{\lfloor c,m\rfloor}=\{\lfloor c,m,j\rfloor=(c,l_{m,j},d_{m,j})$ / $j=1,\ldots,n_m\}$, where $\lfloor c,m\rfloor$ is the coordinate of $D_{\lfloor c,m\rfloor}$ and $\lfloor c,m,j\rfloor$ is the short cut of the coordinate of AE j in $D_{\lfloor c,m\rfloor}$.

Definition 12. Each pair of $\langle z_{c,l,d}|s\rangle$, $\langle(c,l,d)|s\rangle$ or $\langle\lfloor c,m,j\rfloor|s\rangle$ is called Tandem, in short $T\langle z_{c,l,d}|s\rangle$, $T\langle(c,l,d)|s\rangle$ or $T\langle\lfloor c,m,j\rfloor|s\rangle$, if AE $z_{c,l,d}$ or $z_{\lfloor c,m,j\rfloor}$ can be timetabled for subject s.

Definition 13. Realization of $T\langle z_{c,l,d}|s\rangle$, $T\langle(c,l,d)|s\rangle$ or $T\langle\lfloor c,m,j\rfloor|s\rangle$, in short $RT\langle z_{c,l,d}|s\rangle$, $RT\langle(c,l,d)|s\rangle$ or $RT\langle\lfloor c,m,j\rfloor|s\rangle$, is an operation, such that AE $z_{c,l,d}$, or $z_{\lfloor c,m,j\rfloor}$ is in fact timetabled for subject s.

Definition 14. $h_{c,l,d,s}$ or $h_{\lfloor c,m,j\rfloor,s}$ is called subject's height (SH). It is the total number of AE for subject s in APE $Z_{l,d}$ or $Z_{[m,j]}$ plus ($X_{c,l,d}$-1) and its value must be found by the following way(see Appendix):

$$h_{c,l,d,s}=h_{\lfloor c,m,j\rfloor,s}=SH(T\langle(c,l,d)|s\rangle)=SH(T\langle\lfloor c,m,j\rfloor|s\rangle).$$

B. Renewable Data Variables

The Renewable Data Variables (RDV) are completely involved in implementation of the PCA, and define the Time Complexity (TC) of the algorithm. For evaluation of TC of the algorithm the possible maximum length of Input Data $N=\max\{N_c,N_t,N_g,N_l,N_d,N_s,N_q\}$is used. Both the description and the TC of each of Renewable Data Variables are represented below.

$A=\{a_{c,s}$ / $c=1,\ldots,N_c$; $s=1,\ldots,N_s\}$ - $a_{c,s}$ is the number of AE of subject s in class c:

$$TC=N_c\cdot N_s\cdot N_l\cdot N_d\cdot N_t\cdot N_g\le O(N^6).$$

$P=\{p_{c,s}=a_{c,s}-QL(c,s)$ / $c=1,\ldots,N_c$; $s=1,\ldots,N_s\}$ - $p_{c,s}$ is the priority of subject s in class c (see Appendix):

$$TC=N_c\cdot N_s\cdot N_t\le O(N^3).$$

$G^S_{c,l,d}=\{s_{c,l,d,j}$ / $j=1,\ldots,X_{c,l,d}\}$ - the group of all $X_{c,l,d}$ subjects accessible to AE $z_{c,l,d}$, where $X_{c,l,d}$ is the height of $z_{c,l,d}$:

$$TC=N_s\cdot N_l\cdot N_d\cdot N_t\cdot N_g\le O(N^5).$$

$G^S_{l,d}=\{G^S_{c,l,d}$ / $c=1,\ldots,N_c\}$ - the set of all subjects' groups accessible to APE $Z_{l,d}$:

$$TC=N_c\cdot N_s\cdot N_l\cdot N_d\cdot N_t\cdot N_g\le O(N^6).$$

$G^{S*}_{c,l,d}=\{s_j=s^*_{c,l,d,j}$ / $j=1,\ldots,x_{c,l,d}\}$ - is the group of all $x_{c,l,d}$ subjects accessible to AE $z_{c,l,d}$ with the smallest priority $p^*_{c,l,d}$ and with the smallest height $h^*_{c,l,d}$. All the subjects are arranged in descending order by lessons:

$$G^{S*}_{c,l,d}\subset G^S_{c,l,d};\ p^*_{c,l,d}=\min\{p_{c,s}: s\in G^S_{c,l,d}\};$$

$$h^*_{c,l,d}=\min\{h_{c,l,d,s}: s\in G^S_{c,l,d}\};$$

$$TC=N_c\cdot N_l\cdot N_d\cdot N_s\le O(N^4).$$

$H=\{ h_{l,d} = \sum_{c=1}^{N_c} X_{c,l,d}$ / $l=1,\ldots,N_l$, $d=1,\ldots,N_d\}$ - $h_{l,d}$ is the height of APE $Z_{l,d}$:

$$TC=N_c \cdot N_l \cdot N_d \le O(N^3).$$

$D=\{D_{[m]}$ / $m=1,\ldots,N_D\}$ - the set of all N_D APD of Z, where m is the code of $D_{[m]}$ of Z:

$$TC=N_c \cdot N_s \cdot N_l \cdot N_d \cdot N_l \cdot N_d \le O(N^6).$$

$D_{[m]}=\{[m,j]$ / $j=1,\ldots,n_m\}$ - the set of all n_m APE of $D_{[m]}$ of Z, such that for any two coordinates $[m,k]\neq[m,j]\in D_{[m]}$ the following equality is true:

$$G^S_{[m,k]}= G^S_{[m,j]};$$

$$TC=N_c \cdot N_s \cdot N_l \cdot N_d \cdot N_l \cdot N_d \le O(N^6).$$

$D_{\lfloor c,m \rfloor}=\{\lfloor c,m,j \rfloor$ / $j=1,\ldots,n_m\}$ - the set of all n_m AE of $D_{\lfloor c,m \rfloor}$ of Z, such that for any fixed values of c and for any two coordinates $\lfloor c,m,k \rfloor \neq \lfloor c,m,j \rfloor \in D_{\lfloor c,m \rfloor}$ the following equalities are true:

$$p^*_{\lfloor c,m,k \rfloor}=p^*_{\lfloor c,m,j \rfloor};$$

$$G^{S*}_{\lfloor c,m,k \rfloor}=G^{S*}_{\lfloor c,m,j \rfloor};\ G^S_{\lfloor c,m,k \rfloor}=G^S_{\lfloor c,m,j \rfloor};$$

$$X_{\lfloor c,m,k \rfloor}=X_{\lfloor c,m,j \rfloor};\ x_{\lfloor c,m,k \rfloor}=x_{\lfloor c,m,j \rfloor};$$

$$TC=N_s \cdot N_l \cdot N_d \cdot N_l \cdot N_d \le O(N^5).$$

$p_{\lfloor c,m \rfloor}= p^*_{\lfloor c,m,1 \rfloor}$ - the current priority of any $D_{\lfloor c,m \rfloor}\in D$:

$$TC=N_c \cdot N_l \cdot N_d \le O(N^3).$$

$P_{\lfloor c,m \rfloor}=p^*_{\lfloor c,m,1 \rfloor}-n_m+y_{\lfloor c,m \rfloor}$ - is the critical priority of any $D_{\lfloor c,m \rfloor}\subset D$, which in advance defines the priority of all subjects $s_1,s_2,\ldots,s_n\in G^{S*}_{\lfloor c,m,1 \rfloor}$ when all n_m AE of $D_{\lfloor c,m \rfloor}$ are timetabled only for those subjects, where $n=x_{\lfloor c,m,1 \rfloor}$ and the value of $y_{\lfloor c,m \rfloor}$ is found by function Y(c,m) (see Appendix):

$$TC=N_c \cdot N_l \cdot N_d \cdot N_l \cdot N_d \le O(N^5).$$

C. Principle of priority for any iteration of PCA

As it was mentioned above, $p_{c,s}=a_{c,s}-QL(c,s)$ is the priority of subject s in class c (see Appendix). This is the most important variable in searching for the solution of GTP. On one hand, it's evident that, if $p_{c,s} < 0$ for any values of c and s, then no solution of GTP exists. That is, the smallest priority $p_{c,s}$ is the best certificate to be certain, that no solution of GTP exists. On the other hand, the timetabling of an AE $z_{c,l,d}$ for subject s with the smallest priority must decrease the height of the corresponding $Z_{l,d}$ as little as it's possible. In this way it will have the minimum impact on further construction of Z.

Hence, it is quite reasonable to timetable all the subjects with the smallest priority and with the smallest height, first. Namely, these are two basic principles of organizing each iteration of PCA.

D. Critical priority $P_{\lfloor c,m \rfloor}$ of class APD $D_{\lfloor c,m \rfloor}$

According to the description of $D_{\lfloor c,m \rfloor}$ for any fixed value of c and for any two coordinates, such that $\lfloor c,m,1 \rfloor \neq \lfloor c,m,k \rfloor$ and $\lfloor c,m,1 \rfloor,\lfloor c,m,k \rfloor \in D_{\lfloor c,m \rfloor}$, the following equality $p^*_{\lfloor c,m,1 \rfloor}=p^*_{\lfloor c,m,k \rfloor}$ is true. In this respect, the critical priority $P_{\lfloor c,m \rfloor}$ of $D_{\lfloor c,m \rfloor}$ is determined by the following way:

$$P_{\lfloor c,m \rfloor}=p^*_{\lfloor c,m,1 \rfloor}-n_m+y_{\lfloor c,m \rfloor}.$$

The value of $y_{\lfloor c,m \rfloor}$ is found by the function Y(c,m) (see Appendix). In the Table 1, for any values of c and m, the list of all subjects of $s_1,s_2,\ldots,s_n\in G^{S*}_{\lfloor c,m,1 \rfloor}$ is represented. It illustrates how the function Y(c,m) does work. Each row has its denotation $(0,1),\ldots,(y-1,y),\ldots,(l^*-1,l^*)$, which is used for definition of the values of entries in that row.

For example, in the row (0,1), the value of the rightmost entry is 1 and the values of the remaining entries are 0, in the row (1,2), the value of the rightmost entry is 2 and the values of the remaining entries are 1 and so on. Similarly, in the row (y-1,y), the value of the rightmost entry is y and the values of the remaining entries are y-1, etc. The function Y(c,m) finds the value of $y_{\lfloor c,m \rfloor}$ by counting exactly n_m entries starting with the row (0,1), from left to right. Keeping the generality, suppose that counting is ended on the row (y-1,y). If Y(c,m) reaches up the rightmost entry of the row (y-1,y) then $y_{\lfloor c,m \rfloor}=y$. If Y(c,m) reaches up one of the remaining entries of the row (y-1,y) then $y_{\lfloor c,m \rfloor}=y-1$.

As regards the critical priority $P_{\lfloor c,m \rfloor}$ of $D_{\lfloor c,m \rfloor}$, it in advance defines the priorities of subjects $s_1,s_2,\ldots,s_n\in G^{S*}_{\lfloor c,m,1 \rfloor}$ when n_m AE of $D_{\lfloor c,m \rfloor}$ are timetabled only for those subjects. The timetabling of n_m AE of $D_{\lfloor c,m \rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S*}_{\lfloor c,m,1 \rfloor}$ is proceeded by rows, starting with the row (0,1), from left to right (see the Table 1). Keeping the generality, suppose that for any fixed values of c and m, the timetabling is ended on the row (y-1,y). Two situations are possible and both must be always analyzed in further considerations:

Situation 1: If the timetabling reaches up the rightmost entry of the row (y-1,y), then the priority of any subject $s_1,s_2,\ldots,s_k\in G^{S*}_{\lfloor c,m,1 \rfloor}$ will be:

$$P_{\lfloor c,m \rfloor}=p^*_{\lfloor c,m,1 \rfloor}-n_m+y.$$

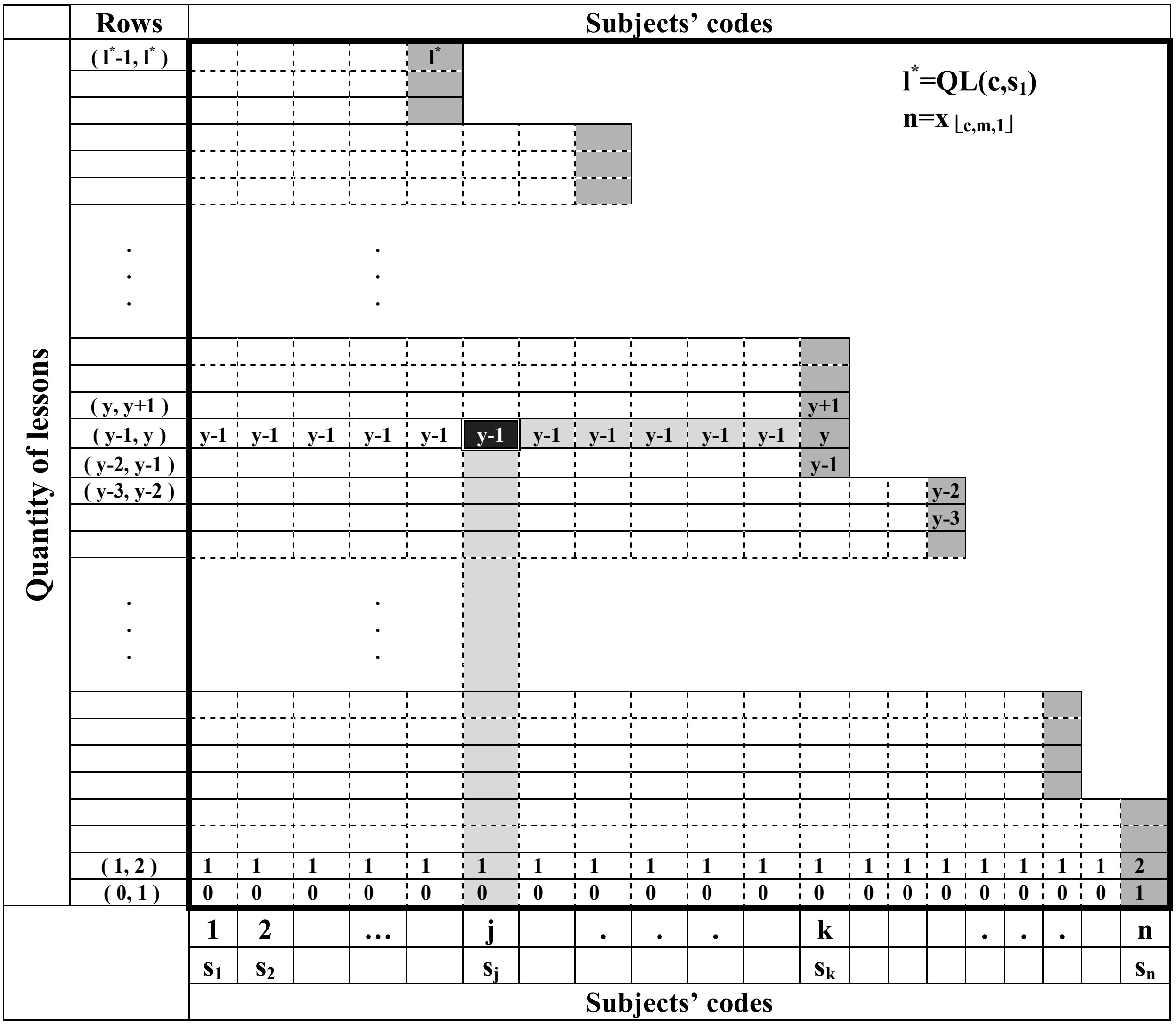
Situation 2: If the timetabling reaches up one of the remaining entries of the row (y-1,y), for example to the entry marked black, then the priority P_R of each subject to the right of the black entry $s_{j+1},s_{j+2},\ldots,s_k\in G^{S*}_{\lfloor c,m,1 \rfloor}$ may be calculated by the following expression:

$$P_R=p^*_{\lfloor c,m,1 \rfloor}-n_m+y-1=P_{\lfloor c,m \rfloor},$$

but the priority of each subject $s_1,s_2,\ldots,s_j\in G^{S*}_{\lfloor c,m,1 \rfloor}$ may be calculated by the following expression:

$$P_L=p^*_{\lfloor c,m,1 \rfloor}-n_m+y=P_{\lfloor c,m \rfloor}+1=P_R+1.$$

Table 1. The list of all subjects $s_1,s_2,\ldots,s_n \in G^{S^*}_{\lfloor c,m,1\rfloor}$, for any fixed values of c and m, which are arranged in descending order by lessons. If $s_1,s_2,\ldots,s_n \in G^{S^*}_{\lfloor c,m,1\rfloor} \subset G^H$, then each subject has the critical priority of $P_{min}=P_{\lfloor c,m\rfloor}$ and the subject's height of $H_{min}=SH\,(T\langle\lfloor c,m,1\rfloor\,|\,s_1\rangle)$.



E. Critical priority, critical height and objective tandems of any iteration of PCA

Definition 15. P_{min} is called the critical priority of iteration (CPI) of PCA. Its value is found by the following way:

$$P_{min}=\min\{P_{\lfloor c,m\rfloor} : \lfloor c,m\rfloor \in D\}.$$

Definition 16. H_{min} is called the critical height of iteration (CHI) of PCA. Its value is found by the following way(see Appendix):

$$G^P=\{G^{S^*}_{\lfloor c,m,1\rfloor} \;/\; \lfloor c,m,1\rfloor \in D_{\lfloor c,m\rfloor} \subset D;\; P_{min}=P_{\lfloor c,m\rfloor}\};$$

$$H_{min}=\min\{SH(T\langle\lfloor c,m,1\rfloor\,|\,s\rangle) : s\in G^{S^*}_{\lfloor c,m,1\rfloor} \subset G^P\}.$$

Definition 17. Each tandem of $T\langle z_{c,l,d}\,|\,s\rangle$, $T\langle(c,l,d)\,|\,s\rangle$ or $T\langle\lfloor c,m,j\rfloor\,|\,s\rangle$, for any fixed values of c,m and s, is called objective tandem (OT) of iteration of PCA and in short $OT\langle z_{c,l,d}\,|\,s\rangle$, $OT\langle(c,l,d)\,|\,s\rangle$ or $OT\langle\lfloor c,m,j\rfloor\,|\,s\rangle$, if $P_{min}=P_{\lfloor c,m\rfloor}$ and

$$H_{min}=SH(T\langle(c,l,d)\,|\,s\rangle)=SH(T\langle\lfloor c,m,j\rfloor\,|\,s\rangle).$$

Definition 18. Realization of $OT\langle z_{c,l,d} | s\rangle$, $OT\langle (c,l,d) | s\rangle$ or $OT\langle \lfloor c,m,j \rfloor \ | s\rangle$ in short $ROT\langle z_{c,l,d} | s\rangle$, $ROT\langle (c,l,d) | s\rangle$ or $ROT\langle \lfloor c,m,j \rfloor | s\rangle$, is an operation, such that AE $z_{c,l,d}$ or $z_{\lfloor c,m,j \rfloor}$ is in fact timetabled for subject s.

Definition 19. The set G^H is called the source of all objective tandems of iteration (SAOTI) of PCA. It is consrtucted by the following way:

$$G^H=\{G^{S^*}{}_{\lfloor c,m,1 \rfloor} \ / \ H_{min}=SH\ (T\langle \lfloor c,m,1 \rfloor \ | s\rangle);\ s\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}\subset G^P\}.$$

Definition 20. G^{OT} is called the set of all objective tandems of iteration of PCA (AOTI). It is constructed using SAOTI G^H by the following way:

$$G^{OT}=\{OT\langle \lfloor c,m,j \rfloor | s_k\rangle \ / \ j=1,2,\ldots,n_m;\ s_k\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}\subset G^H;\ k=1,2,\ldots,x_{\lfloor c,m,1 \rfloor}\},$$

where any subject $s\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}\subset G^H$ has the priority of P_{min} and subject's height of H_{min}.

F. Peaceful coexistence algorithm

Step 0.

Initialize Input Data Variables with Input Data for an instance;
Initialize Output Data Variables with 0;
i = 0; (Comment: i counts the number of iterations)

Step 1.

Determine Renewable Data Variables, taking into account all ROT, if there are any.
If $N_D=0$ then {Timetable of school(Z) is constructed. Print Z ; Goto Step 3;}

Step 2.

$P_{min}=\min\{P_{\lfloor c,m \rfloor} : \lfloor c,m \rfloor\in D\}$;
If $P_{min}<0$ then {Decision problem of GTP does not have solution at all; Goto Step 3;}
$G^P=\{G^{S^*}{}_{\lfloor c,m,1 \rfloor} \ / \lfloor c,m,1 \rfloor\in D_{\lfloor c,m \rfloor}\subset D;\ P_{min}= P_{\lfloor c,m \rfloor}\}$;
$H_{min}=\min\{SH(T\langle \lfloor c,m,1 \rfloor \ | s\rangle) : s\in G^{S^*}{}_{\lfloor c,m,1 \rfloor} \subset G^P\}$;
$G^H=\{G^{S^*}{}_{\lfloor c,m,1 \rfloor}/H_{min}=SH(T\langle \lfloor c,m,1 \rfloor | s\rangle);\ s\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}\subset G^P\}$;
$ROT\langle \lfloor c,m,1 \rfloor | s_1\rangle$;
(Comment: c,m and s_1 are any values such that $OT\langle \lfloor c,m,1 \rfloor | s_1\rangle\in G^{OT}$, see the Table 1)
i=i+1; Goto Step 1;

Step 3.

Stop.

G. Theoretical Foundation of PCA

Remember that PCA realizes in iterations. The main objective of any iteration of PCA is to be certain whether to interrupt or continue the searching for the solution of GTP.

In each iteration, PCA splits Z into class APD using the RDV and calculates their critical priorities. Then PCA finds the value of P_{min} by the following way:

$$P_{min}=\min\{P_{\lfloor c,m \rfloor} : \lfloor c,m \rfloor\in D\}$$

and uses it as CPI. If $P_{min}<0$, then no solution exists, hence the searching for the solution of GTP must be interrupted. If $P_{min}\geq 0$ then the searching for the solution must be continued. In this case, the following two sets $G^P\neq\varnothing$ and $G^H\neq\varnothing$ are created such that $G^H\subset G^P$. The selection of elements of G^P and G^H proceeds in two successive steps by using the variables P_{min} and H_{min} respectively. First of all, the elements of G^P are determined by the following way:

$$G^P=\{G^{S^*}{}_{\lfloor c,m,1 \rfloor} \ / \lfloor c,m,1 \rfloor\in D_{\lfloor c,m \rfloor}\subset D;\ P_{min}= P_{\lfloor c,m \rfloor}\},$$

where P_{min} guaranties that the set G^P will contain all the subjects with CPI. Then PCA finds the value of H_{min} by the following way:

$$H_{min}=\min\{SH(T\langle \lfloor c,m,1 \rfloor \ | s\rangle) : s\in G^{S^*}{}_{\lfloor c,m,1 \rfloor} \subset G^P\}$$

and uses it as CHI. The elements of SAOTI G^H are determined by the following way:

$$G^H=\{\ G^{S^*}{}_{\lfloor c,m,1 \rfloor} \ / \ H_{min}=SH\ (T\langle \lfloor c,m,1 \rfloor \ | s\rangle);\ s\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}\subset G^P\},$$

where H_{min} guaranties that the set G^H will only contain all the subjects with CPI and CHI. For completion of the iteration, only one $ROT\langle (\lfloor c,m,1 \rfloor) | s_1\rangle$ is to be done by PCA, where c,m and s_1 are arbitrary values such that $s_1\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}\subset G^H$ (see the Table 1) and $OT\langle \lfloor c,m,1 \rfloor | s_1\rangle\in G^{OT}$. In this way, the timetable of GTP will be constructed in exactly N_q iterations, if there are any.

For theoretical foundation of PCA we need to prove the following auxiliary lemma.

Lemma: P_{min} of each iteration is the best certificate to be certain that:

if $P_{min}<0$ then no solution of GTP exists and the searching for solution must be interrupted;

if $P_{min}\geq 0$ then the searching for the solution of GTP must be continued.

Proof: It is evident that P_{min} is an integer. So, three cases $P_{min}<0$, $P_{min}=0$ and $P_{min}>0$ are to be distinguished. For each of three cases of P_{min} both Situation 1 and Situation 2 must be analyzed separately. In addition, we must examine the set G^H only, because for any arbitrary values of c and m such that $G^{S^*}{}_{\lfloor c,m,1 \rfloor}\subset G^H$, PCA guaranties that any subject $s_1,s_2,\ldots,s_n\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}$ has the priority of P_{min} and the subject's height of H_{min}. Keeping the generality, suppose that $G^{S^*}{}_{\lfloor c,m,1 \rfloor}$ is the only element in G^H and the timetabling of all n_m AE of $D_{\lfloor c,m \rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}$ is ended on the row (y-1,y) (where $n=x_{\lfloor c,m,1 \rfloor}$, see the Table 1). The timetabling has to be proceeded by rows, starting with the row (0,1). The timetabling of subjects in any row is proceeded from left to right.

Case 1. $P_{min}<0$. We have that $G^{S^*}{}_{\lfloor c,m,1 \rfloor}$ is the only element in G^H. That is, for the values of c and m, the critical priority of each subject $s_1,s_2,\ldots,s_n\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}$ is $P_{\lfloor c,m \rfloor}=P_{min}<0$.

Situation 1: If the timetabling reaches up the rightmost entry of the row (y-1,y), then we have

$$P_{min}=P_{\lfloor c,m \rfloor}=p^*{}_{\lfloor c,m,1 \rfloor}-n_m+y<0.$$

From this, it will follow, that after the timetabling of all n_m AE of $D_{\lfloor c,m \rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}$, the priority of any subject $s_1,s_2,\ldots,s_k\in G^{S^*}{}_{\lfloor c,m,1 \rfloor}$ will be:

$$P_{\lfloor c,m \rfloor}=p^*{}_{\lfloor c,m,1 \rfloor}-n_m+y<0.$$

This indicates that no solution of GTP exists, hence the searching for the solution of GTP must be interrupted.

Situation 2: If the timetabling reaches up one of the remaining entries of the row (y-1,y), for example to the entry marked black, then we have

$$P_{min}=P_{\lfloor c,m\rfloor}=p^*_{\lfloor c,m,1\rfloor}-n_m+y-1<0.$$

From this, it will follow, that after the timetabling of n_m AE of $D_{\lfloor c,m\rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S^*}_{\lfloor c,m,1\rfloor}$, the priority of subjects $s_{j+1},s_{j+2},\ldots,s_k\in G^{S^*}_{\lfloor c,m,1\rfloor}$ will be:

$$P_R=p^*_{\lfloor c,m,1\rfloor}-n_m+y-1<0.$$

This indicates that no solution of GTP exists, hence the searching for the solution of GTP must be interrupted.

Thus, it is proved, that if $P_{min}<0$, then this is the best certificate to be certain that the searching for the solution of GTP must be interrupted.

Case 2. $P_{min}=0$. We have that $G^{S^*}_{\lfloor c,m,1\rfloor}$ is the only element in G^H. That is, for the values of c and m, the critical priority of each subject $s_1,s_2,\ldots,s_n\in G^{S^*}_{\lfloor c,m,1\rfloor}$ is $P_{\lfloor c,m\rfloor}=P_{min}=0$.

Situation 1: If the timetabling reaches up the rightmost entry of the row (y-1,y), then we have

$$P_{min}=P_{\lfloor c,m\rfloor}=p^*_{\lfloor c,m,1\rfloor}-n_m+y=0.$$

From this, it will follow, that after the timetabling of n_m AE of $D_{\lfloor c,m\rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S^*}_{\lfloor c,m,1\rfloor}$, the priority of any subject $s_1,s_2,\ldots,s_k\in G^{S^*}_{\lfloor c,m,1\rfloor}$ will be:

$$P_{\lfloor c,m\rfloor}=p^*_{\lfloor c,m,1\rfloor}-n_m+y=0.$$

and this means that the searching for the solution of GTP must be continued.

Situation 2: If the timetabling reaches up one of the remaining entries of the row (y-1,y), for example to the entry marked black, then we have

$$P_{min}=P_{\lfloor c,m\rfloor}=p^*_{\lfloor c,m,1\rfloor}-n_m+y-1=0.$$

From this, it will follow, that after the timetabling of n_m AE of $D_{\lfloor c,m\rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S^*}_{\lfloor c,m,1\rfloor}$, the priority of subjects $s_{j+1},s_{j+2},\ldots,s_k\in G^{S^*}_{\lfloor c,m,1\rfloor}$ will be:

$$P_R=p^*_{\lfloor c,m,1\rfloor}-n_m+y-1=0$$

and the priority of each subject $s_1,s_2,\ldots,s_j\in G^{S^*}_{\lfloor c,m,1\rfloor}$ will be:

$$P_L=p^*_{\lfloor c,m,1\rfloor}-n_m+y=P_{\lfloor c,m\rfloor}+1>0.$$

and this means that the searching for the solution of GTP must be continued.

Thus, it is proved, that if $P_{min}=0$, then this is the best certificate to be certain that the searching for the solution of GTP must be continued.

Case 3. $P_{min}>0$. We have that $G^{S^*}_{\lfloor c,m,1\rfloor}$ is the only element in G^H. That is, for the values of c and m, the critical priority of each subject $s_1,s_2,\ldots,s_n\in G^{S^*}_{\lfloor c,m,1\rfloor}$ is $P_{\lfloor c,m\rfloor}=P_{min}>0$.

Situation 1: If the timetabling reaches up the rightmost entry of the row (y-1,y), then we have

$$P_{min}=P_{\lfloor c,m\rfloor}=p^*_{\lfloor c,m,1\rfloor}-n_m+y>0.$$

From this, it will follow, that after the timetabling of n_m AE of $D_{\lfloor c,m\rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S^*}_{\lfloor c,m,1\rfloor}$, the priority of any subject $s_1,s_2,\ldots,s_k\in G^{S^*}_{\lfloor c,m,1\rfloor}$ will be:

$$P_{\lfloor c,m\rfloor}=p^*_{\lfloor c,m,1\rfloor}-n_m+y>0.$$

and this means that the searching for the solution of GTP must be continued.

Situation 2: If the timetabling reaches up one of the remaining entries of the row (y-1,y), for example to the entry marked black, then we have

$$P_{min}=P_{\lfloor c,m\rfloor}=p^*_{\lfloor c,m,1\rfloor}-n_m+y-1>0.$$

From this, it will follow, that after the timetabling of n_m AE of $D_{\lfloor c,m\rfloor}$ only for subjects $s_1,s_2,\ldots,s_n\in G^{S^*}_{\lfloor c,m,1\rfloor}$, the priority of subjects $s_{j+1},s_{j+2},\ldots,s_k\in G^{S^*}_{\lfloor c,m,1\rfloor}$ will be:

$$P_R=p^*_{\lfloor c,m,1\rfloor}-n_m+y-1>0$$

and the priority of each subject $s_1,s_2,\ldots,s_j\in G^{S^*}_{\lfloor c,m,1\rfloor}$ will be:

$$P_L=p^*_{\lfloor c,m,1\rfloor}-n_m+y=P_{\lfloor c,m\rfloor}+1>0.$$

and this means that the searching for the solution of GTP must be continued.

Thus, it is proved, that if $P_{min}\geq 0$, then this is the best certificate to be certain that the searching for the solution of GTP must be continued.

The proof of the lemma is completed.

Thus, the following theorem is the theoretical foundation of PCA:

Theorem: The decision problem of GTP has at least one solution iff the CPI of each of N_q iterations $P_{min}\geq 0$ when $ROT\langle\lfloor c,m,1\rfloor | s_1\rangle$, where c, m and s_1 are any fixed values, such that $s_1\in G^{S^*}_{\lfloor c,m,1\rfloor}\subset G^H$ and $OT\langle\lfloor c,m,1\rfloor | s_1\rangle\in G^{OT}$ (see Table 1).

Proof*:* Suppose that the decision problem of GTP has at least one solution and we must prove that the CPI of each of N_q iterations $P_{min}\geq 0$ when $ROT\langle\lfloor c,m,1\rfloor | s_1\rangle$. Indeed, if the decision problem of GTP has at least one solution then from this it will follow that the CPI of the first iteration $P_{min}\geq 0$. According to the lemma the searching for the solution of GTP must be continued. For completion of the first iteration, only one $ROT\langle\lfloor c,m,1\rfloor | s_1\rangle$ is to be done by PCA, where c, m and s_1 are arbitrary values such that $s_1\in G^{S^*}_{\lfloor c,m,1\rfloor}\subset G^H$ (see the Table 1) and $OT\langle\lfloor c,m,1\rfloor | s_1\rangle\in G^{OT}$. Thus, the PCA guaranties that $ROT\langle\lfloor c,m,1\rfloor | s_1\rangle$ will have the minimum impact on further construction of Z, because subject s_1 has the CPI of P_{min} and the CHI of H_{min}. Since the decision problem of GTP has at least one solution then from this it will follow that the CPI of the second iteration $P_{min}\geq 0$. According to the lemma the searching for the solution of GTP must be continued. For completion of the second iteration, only one $ROT\langle\lfloor c,m,1\rfloor | s_1\rangle$ is to be done by PCA, where c, m and s_1 are arbitrary values such that $s_1\in G^{S^*}_{\lfloor c,m,1\rfloor}\subset G^H$ and $OT\langle\lfloor c,m,1\rfloor | s_1\rangle\in G^{OT}$ (see the Table 1). Thus, the PCA again guaranties that

$ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ will have the minimum impact on further construction of Z, because subject s_1 has the CPI of P_{min} and the CHI of H_{min}. It's evident that these observations are true also for each of the next N_q-2 iterations of PCA. It's proved that if the decision problem of GTP has at least one solution then the CPI of each of N_q iterations $P_{min}\geq 0$ when $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$.

For the opposite direction, suppose that the CPI of each of N_q iterations $P_{min}\geq 0$ when $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ and we must prove that the decision problem of GTP has at least one solution. Indeed, if the CPI of each of N_q iterations $P_{min}\geq 0$ when $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ it will follow that the CPI of the first iteration $P_{min}\geq 0$. According to the lemma the searching for the solution of GTP must be continued. For completion of the first iteration, only one $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ is to be done by PCA, where c, m and s_1 are arbitrary values such that $s_1\in G^{S*}{}_{\lfloor c,m,1\rfloor}\subset G^H$ (see the Table 1) and $OT\langle\lfloor c,m,1\rfloor|s_1\rangle\in G^{OT}$. Thus, the PCA guaranties that $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ will have the minimum impact on further construction of Z, because subject s_1 has the CPI of P_{min} and the CHI of H_{min}. Since the CPI of each of N_q iterations $P_{min}\geq 0$ when $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ it will follow that the CPI of the second iteration $P_{min}\geq 0$. According to the lemma the searching for the solution of GTP must be continued. For completion of the second iteration, only one $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ is to be done by PCA, where c, m and s_1 are arbitrary values such that $s_1\in G^{S*}{}_{\lfloor c,m,1\rfloor}\subset G^H$ and $OT\langle\lfloor c,m,1\rfloor|s_1\rangle\in G^{OT}$ (see the Table 1). Thus, the PCA again guaranties that $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ will have the minimum impact on further construction of Z, because subject s_1 has the CPI of P_{min} and the CHI of H_{min}. It's evident that these observations are true also for each of the next N_q-2 iterations of PCA and one of the solutions of GTP will be constructed in exactly N_q iterations. It's proved that if the CPI of each of N_q iterations $P_{min}\geq 0$ when $ROT\langle\lfloor c,m,1\rfloor|s_1\rangle$ then the decision problem of GTP has at least one solution.

The proof of the theorem is completed.

H. Time Complexity of PCA

So, it is evident that TC of PCA is $\leq O(N^7)$.

IV. CONCLUSION

P=NP.

V. RESULT OF PCA

Using PCA a computer program was developed in C++. As an instance of implementation of GTP the Input Data of a certain school is processed. For this instance the program consumed 73 seconds on IBM PC with a processor P-VI 3000 for constructing the timetable.

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APPENDIX

Auxiliary Functions:

ceil(k)- returns the minimum integer that is greater than or equal to k.

floor(k)- returns the maximum integer that is less than or equal to k.

$$\{condition\ ?\ 1\ :\ 0\} = \begin{cases} 1, & \text{if condition holds;} \\ 0, & \text{if not;} \end{cases}$$

T(c,l,d)- returns the set of teachers' codes which teach subject $z_{c,l,d}$ in class c at hour l on day d, if $z_{c,l,d}\neq 0$;
returns $\varnothing$, if $z_{c,l,d}=0$.

ShT(c,t,s)- returns the set of teachers' codes which teach subject s in class c, if $q_{c,t,s}\neq 0$;
returns $\varnothing$, if $q_{c,t,s}=0$.

QL(c,s)- returns the quantity of lessons of subject s in class c.

IsSh(c,s)- returns 1, if s is a Sharing subject in class c;
returns 0, otherwise.

$SH(T\langle(c,l,d)|s\rangle)$- returns the total number of AE for subject s in APE $Z_{l,d}$ plus $(X_{c,l,d}-1)$.

Y(c,m)- returns the value of $y_{\lfloor c,m\rfloor}$. The listing of the function is provided below:

```
Y(int c, int m)
{
int ne=n_m, fl=0, yr, y=0, n=x⌊c,m,1⌋;
for (int i = n; i >= 1; i--)
{
s=s_i;      ql=QL(c,s);       (Comment: see the Table 1)
if (fl == ql)        continue;
yr = ql - fl;
      for (int  j = yr; j >= 1; j--)
      {
            if (ne  <  i)  return y;
            ne = ne - i; y++;
      }
fl=ql;
}
return y;
}
```